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A Novel Approach for Designing A Feedback Controller of linear Time Invariant Networked Control Systems with Delayed-Transmission Time

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Abstract — This paper investigates the problem of stabilization processes of networked control systems (NCSs) with delayed-transmission time. The investigation deals with control problem of linear time invariant (LTI) of NCSs when the plant and the controller belong to the same network. Long time delays due to the transmission element may degrade and destroy the stability of a networked control system. To overcome this problem a new exact and novel approach is analytically obtained and the delay elements in system variables have been augmented and moved to the systems parameters. An output control feedback is introduced for designing a controller based on using the same conventional control technique in the literature. A design procedure for stabilizing the linearized model of NCSs involving a time delay based on the alternative generalized model is introduced. As a result, the effect of the delay factor is completely eliminated from the system's variables and moved to the systems parameters. The design procedure of the controller that moves the finite eigenvalues of the system to arbitrary locations simultaneously is carried out in a manner similar to those obtained for non-delayed conventional state space systems. The coefficients of the feedback control law can be easily evaluated which makes it possible to update the controller's parameters on-line with the change of the operating point. It is shown that the non-delayed ordinary state space systems appear as special cases of the present work when the delay elements vanish. To support and illustrate the effectiveness and usefulness of the work presented for the proposed technique an example based on the transformed model derived in this work is introduced.

Index Terms — networked control systems (NCSs), stability of a network, time delays, generalized model, controller's parameters

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1 Introduction

Recently, many works have been focused on feedback control systems wherein the control loops are closed through a real-time network which are called networked control systems(NCSs) [2,4-8]. The control systems of NCSs are those systems that use the well known classical closed loops in their communication networks where all information exchanged in the networks are subjected to have time delays induced by the shared medium (plant – to - controller delay and controller-to-plant delay). Many known examples of NCSs can b e found in several articles, especially in the field of automated manufacturing systems, mechatronic, robot manufacturing systems, mechatronic, robot systems, and teleoperation systems [2]. This kind of networks that induce delay signals in their operations (plant-to-controller delay and controller-to- plant delay) have been treated with various approaches in the literature. Several other works have focused on stabilization processes of NCSs. The author in [9] studied the effect of delays on the system modeling and then a new optimal controller was designed to control the plant using an approximate technique. Clock synchronization approach has been utilized to evaluate the delays online in [9] with the implementation of the controller caused some performance degradation. Another view to tackle the delays problem is presented in [10] by using a fuzzy logic controller to control the NCSs, which ended up with no use of the communication information in design of controller. Since networked control systems is an integrated research area, which is not only concerned about control, but also relevant to communication, we must combine the knowledge of control and communication together to improve the system performance.

It is known that in NCSs each components of the control system, such as sensors, controllers, actuators, etc. are connected via real-time network. Fig.1. shows information flow (reference input, plant output, control input, etc.) is exchanged through the

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network. However, the insertion of the network in the feedback control loop makes the analysis and design of an NCS complex because the network imposes an undetermined communication delay. Therefore, conventional control theories with many ideal assumptions, such as synchronized control and non-delayed sensing and actuation, must be reevaluated before they can be applied to NCSs. In recent literature , many authors have concluded that the enormous advantages of NCSs are their lower cost and their ability of reducing weight and power, simpler installation, easing of system diagnosis, increasing system edibility, and reducing system wiring and maintenance with increasing system agility [2]. On the other hand, the NCSs possesses a drawback that exited from the augmented complexity of analysis and design with respect to conventional feedback control systems [2]. However, the insertion of the network in the feedback control loop makes the analysis and design of an NCSs complex because the network imposes an undetermined communication delay element. Therefore, most conventional control theories can not be applied without making some modifications and reevaluation before they can be applied to NCSs. The essential issues need to be addressed in NCSs are the network-induced delays, which either constant or time varying, can degrade the performance of control systems and can even destabilize the system. For this reason, there have been a lot of researches on NCSs focused on reducing the performance degradation caused by the delays factors. The presence of these delays elements which could be either constant or time-varying, can disturb the performance of control systems designed when their effect are ignored or disregarded in the controller's design which could also lead to even destabilize the system [2]. Most authors handled out these delays phenomena with some approximate techniques which give no exact information about the system behaviors.

In [8] the sensor is separated from the controller and the approach ended with two scheduling methods, including try once discard (TOD) and a statically. scheduled for providing the system stability is guaranteed . In [2] an analytical technique is introduced for simple cases while a simulation processes is used in the case of having complex situations. another author [11] focused on the design of network controllers based on how controlled communication systems address the expected delays . The tracking control with time delay compensation for NCSs has been introduced by Claudio et al in [2]. Their approach is mainly based on putting some sever restrictions on the system parameters such as the zeros of the plant transfer function and the eigenvalues of the exosystem dynamic matrix do not have values in common as well as the input to the system must satisfy u(t) = 0, for all t < 0. These restrictions make their method so special and can be applied only on some especial cases that satisfy the assumed restrictions introduced in their work.

In the rest of the paper is organized as follows. In Section II, a continuous model for a class of networked control systems (NCSs) with delayed-transmission time is derived and system description is given. In Section III, A Unique and Exact Alternative Model of NCSs with delayed-transmission time system is obtained and derived conventional generalized state space systems and a design of a controller to stabilize NCSs are investigated. Section V numerical example is presented to support the robustness of our work and to illustrate that our proposed methods are less conservative and more effective. Finally, the conclusions are given in Section IV.

2 PROBLEM STATEMENT

Let us consider a plant (P) connected to a network (N) and a controller (W) as shown in Fig.1 are linear time invariant (LTI) systems representing two

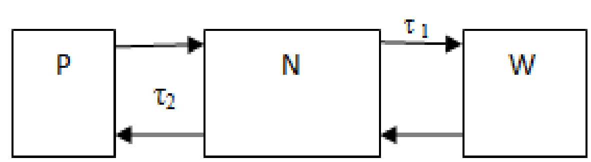

Fig.1. Networked Control Systems with Delayed-Transmission Time

nodes of the same network with time delays in the forward transmission to the controller is τ_1 and with time delays in the feedback transmission from network to the plant is τ_2 as shown Fig.1. These time delays, precisely are assumed to be known and constant, but not necessarily equal. The dynamical description for the plant P takes the form

$$\dot{x}(t) = A\,x(t) + B\,u(t)\,, \qquad (1\text{-a})$$
$$y(t) = C\,x(t)\,, \qquad (1\text{-b})$$

where $x \in R^n$, u, $y \in R^n$ and A's,B,C are real matrices of appropriate dimensions. It is well known that due to the presence of the network, the plant output y(t) signal reaches the controller side as input with delayed-transmission time as shown in (Fig.1), is given by

$$u_w(t)=C\,x(t-\tau_1) \qquad (2)$$

It is an easy task to see that the feedback controller signal reaches the plant with an output feedback control law such that the eigenvalues of the closed-loop system have negative real part and the plant state variables x(t) reach their stable states asymptotical, which is possibly generated by

$$u(t)=k\,C\,w(t-\tau_2) \qquad (3)$$

where k is the controller's gain.

Substituting (2) and (3) in (1), yields

$$\dot{x}(t)=Ax(t)+A_T\,x(t-\tau), \qquad (4)$$

Where $\tau = \tau_1 + \tau_2$ and $A_T = B\,k\,C$

3 A Unique and Exact Alternative Model of System (4)

This section outline the technique introduced by Saidahmed [12] used in converting systems of the form (4) into a unique and exact alternative conventional finite dimensional model whose dynamical description has no delays in the states nor in the control. As will be seen shortly, this model contains system (1) as a special case when the delay parameter goes to zero. The next theorem shows the idea behind our approach of finding a criteria for moving the delay the parameter τ from the state variables to be augmented into the system parameters.

Theorem1: For the feedback linear time-invariant system with delay elements in the states described by (4) , there always exit a linear transformation that moves the delays element from the state variables to the system parameters of the form

$$\dot{x}(t) = A\,x(t) \qquad \text{for } 0 \leq t \leq \tau \qquad (5\text{-a})$$

And

$$T(\tau)\dot{x}(t) = \hat{A}\,x(t) \quad \text{for } t \geq \tau \qquad (5\text{-b})$$

Where (5-b) is a unique and exact alternative model in the form of generalized state space system,

$T(\tau) = I + A_T\,A(\tau)$, $\hat{A} = A + A_T$

and

$$A(\tau) = \int_0^{\tau} e^{-A\theta} d\theta$$

***Proof*:** We prove this theorem by introducing the linear transformation introduced by saidahmed [12] of the form

$w(\tau,s) = e^{\tau s}x(s)$ with initial value $w(0,s)$ is

$$w(0,s) = x(s) \qquad (6)$$

By taking Laplace transform of (4) and applying the linear transformation given in (6), results in

$$\frac{dw(\tau,s)}{d\tau} = A\,w(\tau,s) + e^{\tau s}x_0 + A_T x(s) \qquad (7)$$

Solving (7) with respect to $w(\tau,s)$, yields

$$w(\tau,s) = e^{A\tau}x(s) + \left[\int_0^{\tau} e^{A(\tau-\theta)}d\theta\right]A_T x(s) + \left[\int_0^{\tau} e^{A(\tau-\theta)}e^{\theta s}d\theta\right]x_0 \qquad (8)$$

It is an easy task that the most right hand term of (8) can be rearranged to have the form

$$\left[\int_0^{\tau} e^{A(\tau-\theta)}e^{\theta s}d\theta\right]x_0 = (sI - A)^{-1} * \left[e^{s\tau}I - e^{A(\tau)}\right]x_0 \qquad (9)$$

Substituting (9) into (8) and, converting the result into the time domain, we obtain

$$x(t) = \left[e^{A\tau} + \int_0^{\tau} e^{A(\tau-\theta)}A_T\,d\theta\right] * x(t-\tau)u_s(t-\tau) + e^{At}x_0\left[u_s(t) - u_s(t-\tau)\right] \qquad (10)$$

where $u_s(\,.\,)$ stands for a unit step function.
It is clear from examining (10) that for $0 \leq t < \tau$, we get

$$x(t) = e^{At}x_0\left[u_s(t) - u_s(t-\tau)\right]$$

and for $t \geq \tau$, we have

$$x(t) = e^{A\tau}x(t-\tau) + \left[\int_0^{\tau} e^{A(\tau-\theta)}d\theta\right] * A_T\,x(t-\tau) \qquad (11)$$

Multiplying both sides of (11) by $e^{-A\tau}$ and substituting the result into (4) and collecting similar terms, results in

$$\left[\int_0^{\tau} e^{-A\theta} d\theta \quad \right] \dot{x}(t) = x(t) - x(t-\tau) \qquad (12)$$

Premultiplying (12) by A_T, using (4) and collecting similar terms, we end up with

$$T(\tau)\dot{x}(t) = \hat{A}\, x(t) \quad \text{for} \quad t \geq \tau \qquad (13)$$

Equation (13) is in the form of linear tine-invariant singular which contains the non-delay system as special case see [15] for more information about stabilizing (13). To see this, let $\tau = 0$ in (13), yields

$$\dot{x}(t) = (A + A_T)x(t) \qquad (14)$$

which shows direct verification of present approach. It should be mentioned that (13) is also a unique alternative representation of (4) in the sense that the behavior of the system is uniquely determined by (13). On the other hand, the dynamical behavior of the system for $0 \leq t < \tau$ with $\tau > 0$, can be obtained also from (11) as

$$\dot{x}(t) = A_1 x(t) + Bu(t), \ 0 \leq t < \tau \qquad (15)$$

as expected. Indeed, we could have been obtained (15) by inspection from (4) by knowing that $A_T x(t-\tau) u_s(t-\tau) = 0$ for $0 \leq t < \tau$. This strengths theorem (1) and supports the idea that (15) describes completely the behavior of the system for $0 \leq t < \tau$. This completes the proof.

It is important to note that in most practical cases the matrix $T(\tau)$ in (13) is invertible and this reduces the difficulty which usually encountered when dealing with states-delay systems. It should also be mentioned that (15) reveals that system (4) is completely controlled during the period $0 \leq t < \tau$, with $\tau > 0$, by the dynamical equation (15) and this is in complete agreement with Walsh approximation method [9] and the time- partition method [l0]. Since system (13) possesses neither delay in the states nor in the control, then it will be considered in this paper as a non-delayed generalized state space system. Using the approach given in [12-14], a reduced order controller for systems of the form (13) can be designed. However, From the generalized theory of singular systems, it is well known [12] that (13) is solvable iff $(A + \zeta T(\tau))^{-1}$ exits for some scalar $\zeta \epsilon R$. where R is a field of real numbers [15].

Using (13) which is considered as a non-delayed state space generalized system. It is an easy task to design an output feedback control law using the approach introduced by saidahmed in [12] such that all the eigenvalues of the closed-loop system can be adjusted to have negative and those with zero real part must be simle for an arbitrary values of k. As a special case of (13) when the matrix $T(\tau)$ is invertible for all values of $\tau \in R_n$, where R is a field of real numbers, then all eigenvalues of the closed loop system shown in Fig.1 can be displaced to arbitrary locations. It important to note that using (13) gives an exact alternative form of (1) with neither delay in the states nor in the control and the delay element has been absolutely moved to the system parameters which means no kind of any approximation has been utilized and all needed paper as a non-delayed generalized state space system. Using the approach given in [12], a reduced order controller for systems of the form (13) can be designed. However, From the generalized theory of singular systems [15], it is well known that (13) is solvable iff $(A + \zeta T(\tau))^{-1}$ exits for some scalar $\zeta \epsilon R$, where R is a field of real numbers. Using (13) which is considered as a non- delayed state space generalized system. it is an easy task to design an output feedback control law using the approach introduced by saidahmed in [15] such that all the eigenvalues of the closed-loop system can be adjusted to have negative and those with real part must be simple for an arbitrary values of k. As a special case of (13) when the matrix $T(\tau)$ is invertible for all values of $\tau \in Rn$, where R is a field of real numbers, then all eigenvalues of the closed loop system shown in Fig.1 can be displaced to arbitrary locations. It important to note that using (13) gives an exact alternative form of (1) with neither delay in the states nor in the control and the delay element has been absolutely moved to the system parameters which means no kind of any approximation has been utilized and all needed behaviors of the system have been completely treated by saidahmed in [12]-[15]. From the generalized theory of singular systems [15].

Example 1: Let a linear time-invariant state-delay system shown in Fig.1 be described by

$$\dot{x}(t) = 1/4\,x(t) + u(t), x(0) = 2.0 \quad (16\text{-a})$$

$$y(t) = x(t) \quad (16\text{-b})$$

With transmission delay $\tau = 1$, and $k = -1$.

It is desired to analyze (16) using theorem (1) and using (5-a) with the given initial conditions $x(0) = 2.0$ for $0 \le t < 1$, we have

$x(t) = x(0) = 2.0$ for $0 \le t < 1$

from (11), the initial value at $\tau = 1$ can be obtained as

$$x(1) = \left[e^{0.25} - \int_0^1 e^{0.25(1-\theta)}\, d\theta \right] = -0.148$$

and from (5-a) for $t \ge 1$, a unique and exact alternative form of (5-b) can be obtained as

$$\dot{x}(t) = -6.5x(t),\ x(1) = -0.148 \quad \text{for } t \ge 1 \quad (17)$$

It is clear from (17) that the unstable networked control systems with delayed-transmission with a controller based on the output feedback has been stabilized by the proper choice of the feedback gain k based on the same technique used with the conventional state space design approach for controlling the LTI systems. This results support the effectiveness of our approach introduced in this paper.

4 CONCLUSION

In this paper, One of the most practical application of the generalized model developed in this paper is the stabilization process of the networked control systems (NCSs) involving a delay in the transmission media, a new linear transformation is obtained that transforms the networked control LTI systems with delayed elements due to transmission networks into a conventional generalized state space system with no delays in the state nor in the control. It was shown that the delays elements in the networked control systems (NCSs) which has been transformed to a finite dimensional linear time - invariant generalized state space system are moved to the system structures with no delays either in the states nor in the control. This new technique guarantees closed-loop asymptotic stability. The stability result also implies that stable feedback of networked control LTI systems with delayed elements are robust and the control law was treated similar to those given in the conventional control systems. An example based on transformed model was also provided.

The major feature of this new model is its ability to serve as a major tool in developing new qualitative properties of linear state-delay systems. It is shown that the transient response of such system can be controlled by using only one easily implemented in a manner analogous to those given to the standard state space linear control systems, using the non-delay state variables. The results obtained are much more direct and the presence of time delay in transmission media does not possess any problem in designing the controller compared with those given by the tracking control with time delay compensation, finite spectrum assignment method [2], and the edge theorem [4], mainly because the present design procedure allows possible updating of the controller's parameters on-line with the change of the operating point. It should be also emphasized that the generalized alternative model (5) investigated in this paper is expected to have a wide range of practical applications, especially in the field of peer to peer protocols systems.

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